Eccentric Elliptical Contours in Total Hip Replacements

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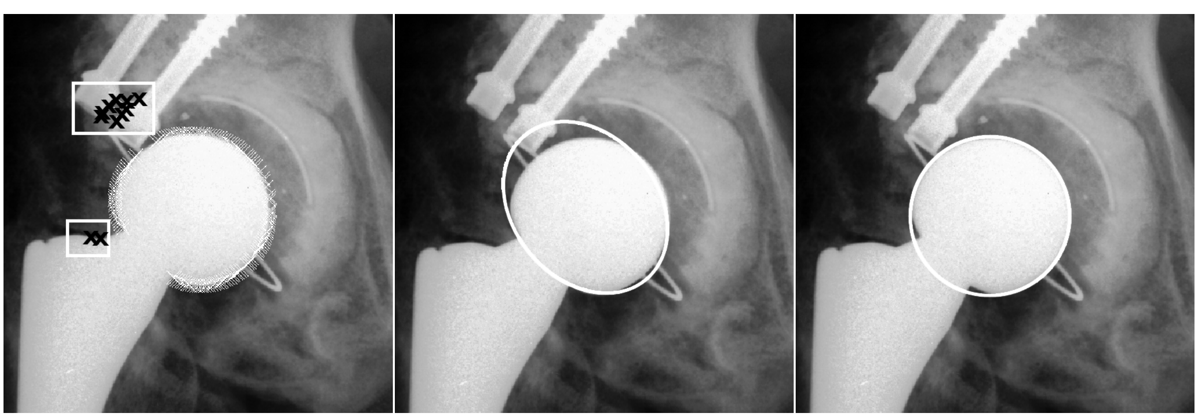
Abstract. The active ellipses method for assessing wear in total hip replacements uses robust ellipse fitting to localise the contours of the femoral head and acetabular rim wire marker. In the case of the latter these ellipses can be very eccentric and the standard algebraic distance was shown to be inadequate. The geometric distance from an ellipse to a point is not trivial to compute and thus numerous error of fit functions have been created. In this work several of these error of fit functions are compared, including a geometric error of fit function, on both synthetic data and by using active ellipses on a set of test radiographs containing eccentric rims.
Least squares estimation using a geometric error function was most accurate in the presence of Gaussian noise. However, least median of squares estimation using a geometric error function was most accurate in the presence of outliers. Furthermore, its performance was similar to that of a computationally cheaper error function known as the foci bisector distance to the extent that the two were almost interchangeable.

1 Introduction

The active ellipses method for assessing wear in total hip replacements (THRs) uses robust ellipse fitting to localise the femoral head and acetabular rim in radiographic images [1]. Radiopaque clutter, such as seen in Figure 1(a) causes structured outlying points from which a standard least squares (LS) ellipse fit [2] generates erroneous results, as seen in Figure 1(b). A robust method such as least median of squares (LMedS) is desirable in this instance as it has a breakdown point of 50% outlying points (see Figure 1(c)). Most conventional ellipse fitting uses an algebraic error function but this can cause problems when fitting eccentric ellipses such as those of the acetabular rim. However, other error of fit functions can be used, the most obvious choice being an error based on the geometric distance of a data point from the closest point on the ellipse curve. This distance is not trivial to compute and thus numerous computationally cheaper error functions have been considered in the past [3, 4]. Today's computational power and the availability of an efficient algorithm [5] have increased the feasibility of using the geometric distance.

This paper reports an empirical comparison of the performance of LS and LMedS fitting with algebraic and geometric error functions using synthetic data. Additionally, the performance of LMedS with weighted algebraic and foci bisector distance error functions was evaluated. Finally, the best performing fitting algorithms were used to localise the elliptical structures in THR radiographs.



(a) (b) (c)

Figure 1. (a) Data points found during a femoral head search using an active ellipse. White crosses denote inliers. Outliers are shown in black and are highlighted by white rectangles. (b) An LS fit to the data points. (c) A robust LMedS fit which finds a good solution in the presence of outliers.

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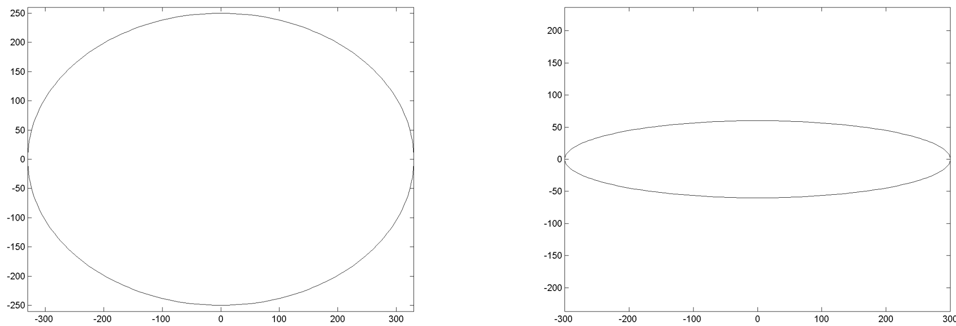
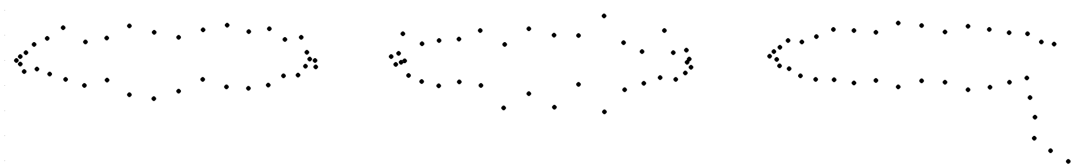


(a) (b)

Figure 2. The two ellipses used to generate synthetic data. Both were centred at the origin and aligned with the image axes. (a) The less eccentric ellipse, $a = 333$, $b = 250$. (b) The more eccentric ellipse, $a = 300$, $b = 60$.



(a) (b) (c)

Figure 3. Data points created from the more eccentric ellipse. (a) Gaussian noise ($\sigma = 10$). (b) Half low variance Gaussian noise ($\sigma = 5$) and half high variance Gaussian noise ($\sigma = 20$ in this visualisation). (c) Structured outliers sampled from noisy line segments.

2 Materials and methods

2.1 Experiments using synthetic data

In order to assess the performance of the ellipse fitting algorithms, synthetic, noisy data sets were created from known ellipse parameters. Two ellipses were considered with eccentricities of 0.66 and 0.98 (see Figure 2). Eccentricity is defined as $\sqrt{1 - \frac{b^2}{a^2}}$ where a is the major semi-axis and b is the minor semi-axis of the ellipse. Three types of data set were created from each of the two ellipses.

Gaussian noise Data points were sampled at uniform intervals along the ellipse with additive Gaussian noise in the direction normal to the ellipse contour. Data sets were created with $\sigma = 0$, 10, 20, 30, 40, 50, 60 and 70.

Gaussian outliers Some points were sampled with noise drawn from a low variance Gaussian ($\sigma = 5$) while the remainder had high variance Gaussian noise ($\sigma = 500$) thus creating outlying points. Data sets were created with the percentage of points with high variance noise set to 0%, 10%, ..., 90%.

Structured outliers This type of data set was designed to simulate structured noise by sampling some of the points from straight line segments close to the ellipse. All points were sampled with Gaussian noise ($\sigma = 5$) from a closed contour, 80% from an elliptical arc, 10% from a line segment orthogonal to that arc, and 10% from another line segment rotated $45°$ with respect to the first line segment. Data sets were created with the percentage of structured outliers set to 0%, 10%, ..., 90%.

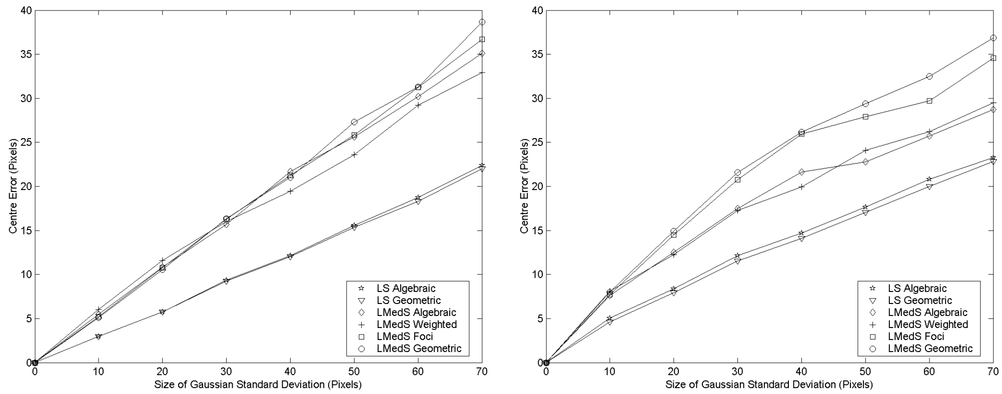
An example from each type of data set is shown in Figure 3. Each example consisted of 38 points. LS fits using geometric and algebraic error functions and LMedS fits (with LS fine tuning on resulting inliers) using algebraic, weighted algebraic by gradient [4], foci bisector distance [4] and geometric error [5] functions were performed. The Euclidean distances between the original and recovered centre points were used as a measure of accuracy.

2.2 Experiments using radiographic data

The elliptical projections of acetabular rims in standard clinical radiographs typically have eccentricities between 0.8 and 1.0. A set of 19 radiographs containing Zimmer CPT prostheses with particularly eccentric rim projections (> 0.96) was obtained. The most accurate error-of-fit functions from the experiments using the synthetic data sets described above were selected. These were used to perform active ellipse localisation on the radiographs using robust LMedS fitting. Localisation was run twice on each image, providing 38 results per method. No LS fine tuning was performed on the resulting inliers.

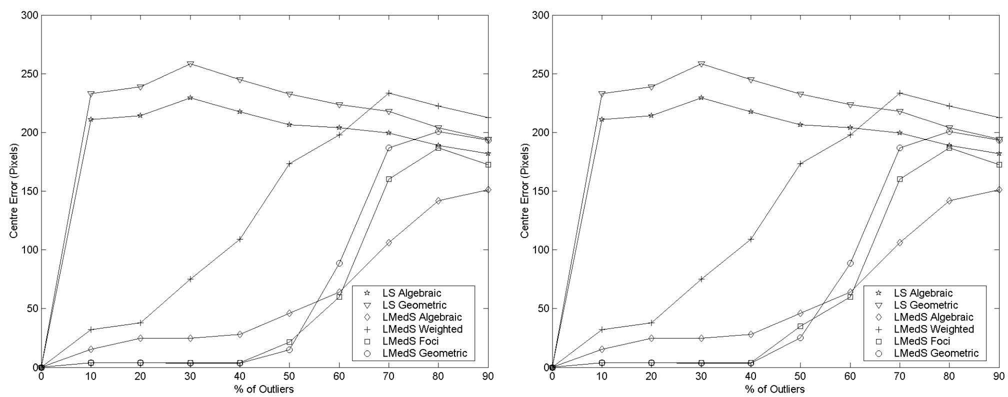
3 Results

Figures 4-6 show the alpha trimmed means ($\alpha = 0.1$) of the centre errors for each of the synthetic data sets. Each point on these plots was computed from 500 examples. In Figure 6 the centre error goes beyond the scale of the graph as selecting a large proportion of points from the clutter resulted in extremely eccentric and erroneous ellipses being generated.



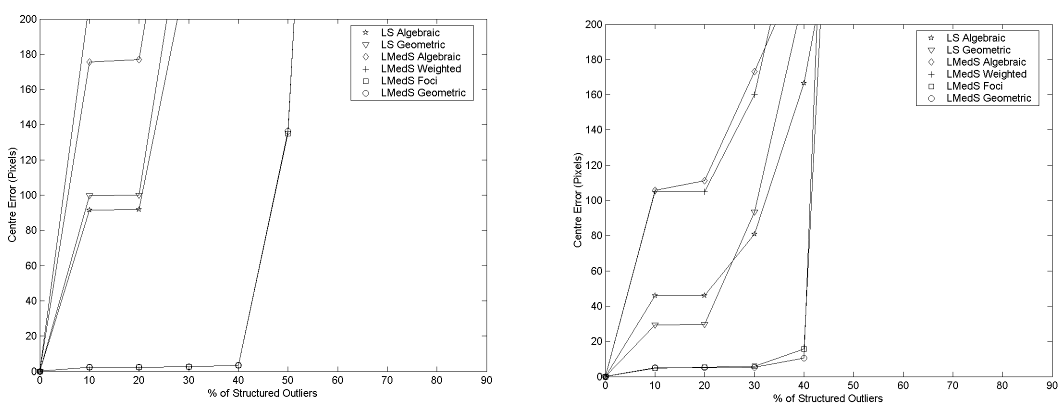
(a) (b)

Figure 4. Centre errors for (a) less eccentric and (b) more eccentric synthetic ellipse data as a function of σ.



(a) (b)

Figure 5. Centre errors for (a) less eccentric and (b) more eccentric synthetic ellipse data with Gaussian outliers.

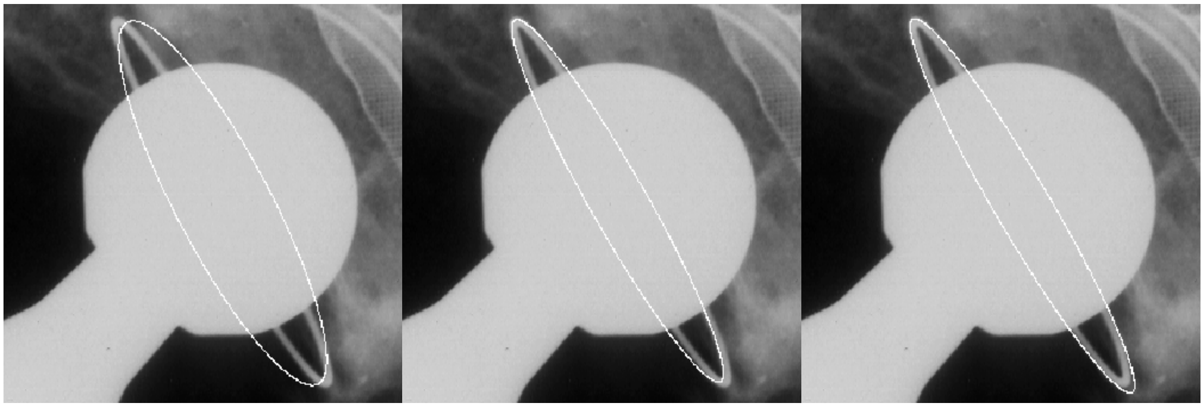


(a) (b)

Figure 6. Centre errors for (a) less eccentric and (b) more eccentric synthetic ellipse data with structured outliers.

On the radiograph dataset, geometric fitting failed 15 times out of 38, foci bisector fitting failed 16 times and algebraic fitting failed 32 times. Given the difficulty of the data set and the absence of any LS fine tuning to inliers identified from the minimal subset (which increases the performance of all three error functions), the results using

LMedS geometric fitting were encouraging. An example of the output of each of these algorithms is shown in Figure 7.



(a) (b) (c)

Figure 7. An eccentric rim with (a) failed algebraic, (b) successful foci bisector and (c) successful geometric fits.

4 Discussion

In the presence of pure Gaussian noise, LS outperformed LMedS irrespective of the error-of-fit function used (see Figure 4). LS using the geometric error performed the best. Least squares fitting is based on an assumption of Gaussian noise and is optimal under these circumstances. On the more eccentric ellipse the difference between LS algebraic and LS geometric became more pronounced, with LS geometric performing better. The LMedS geometric and LMedS foci bisector methods were least accurate in this case.

However, the robust LMedS fitting was more accurate in the case of outliers, whether Gaussian or structured. The plots in Figures 5 and 6 show that LMedS using foci bisector or geometric error-of-fit functions performed best on both ellipse eccentricities. The only noticeable difference between these two methods before the breakdown point was at 40% structured outliers in Figure 6(b). Results obtained with greater than 50% outliers lie beyond the theoretical break-down point of LMedS and so not surprisingly are poor. LMedS fitting with algebraic and or weighted algebraic functions performed very poorly in the presence of structured noise (see Figure 6). In fact LS methods were better in this case as LS fitted to both the straight line segments and the elliptical arc, while LMedS algebraic and weighted algebraic tended to favour points on the line segments.

The LMedS methods using geometric, foci bisector and algebraic functions were compared on the radiographic data. The latter was included because it has been used previously for this application. LMedS geometric made the most successful estimates, just outperforming LMedS foci bisector. LMedS algebraic performed poorly.

These experiments demonstrated that in the presence of non-Gaussian noise, LMedS geometric fitting tended to perform slightly better than the foci bisector distance approximation. However, it is a computationally expensive method and in applications where speed is important the foci bisector distance is recommended as an error of fit function. The authors are not aware of any previous work in the literature using the geometric distance [5] as an error function for LMedS ellipse fitting.

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References

1. S. Kerrigan, S. J. McKenna, I. W. Ricketts et al. “Analysis of total hip replacements using active ellipses.” *Medical Image Understanding and Analysis* pp. 177–180, 2003.
2. A. Fitzgibbon, M. Pilu & R. Fisher. “Direct least square fitting of ellipses.” *IEEE Transactions on Pattern Analysis and Machine Intelligence* **21(5)**, pp. 476–480, 1999.
3. P. Rosin. “Assessing error of fit functions for ellipses.” *Graphical models and image processing* **58(5)**, pp. 494–502, 1996.
4. P. Rosin. “Analyzing error of fit functions for ellipses.” *Pattern Recognition Letters* **17**, pp. 1461–1470, 1996.
5. S. J. Ahn, W. Rauh & H. J. Warnecke. “Least squares orthogonal distances fitting of circle, sphere, ellipse, hyperbola and parabola.” *Pattern Recognition* **34**, pp. 2283–2303, 2001.