Affine Invariant Texture Analysis Based on Structural Properties[1]

Jianguo Zhang, Tieniu Tan
National Laboratory of Pattern Recognition (NLPR),
Institute of Automation, Chinese Academy of Sciences, Beijing, 100080, P.R.China
{jgzhang, tnt} @nlpr.ia.ac.cn

Abstract

This paper presents a new texture analysis method based on structural properties. The texture features extracted using this algorithm are invariant to affine transform (including rotation, translation, scaling, and skewing). Affine invariant structural properties are derived based on texel areas. An area-ratio map utilizing these properties is introduced to characterize texture images. Histogram based on this map is constructed for texture classification. Efficiency of this algorithm for affine invariant texture classification is demonstrated using Brodatz textures.

1. Introduction

Texture analysis is a fundamental issue in computer vision and pattern recognition. Existing texture analysis methods can be broadly split into three categories: statistical method, model based methods, and structural methods [1]. In many practical applications, we cannot ensure that texture images are acquired from the same viewpoint and usually these images undergo affine or perspective distortion. Texture analysis methods invariant to such distortion are highly desirable. More and more efforts have been devoted to this important issue. However most of the existing invariant texture analysis methods only focus on rotation, translation, and scale transform. Survey papers of the work on this important field may be found in [2][7]. Little work can be found on affine or perspective invariant texture analysis [2][12]. Some new algorithms on affine pattern analysis have been proposed recently [7][8][9][10][11]. Whether these methods could be used for affine invariant texture analysis requires further investigation.

Structural properties of texture elements such as compactness area and perimeter have been successfully used to character texture image [3][14]. Some invariant texture analysis methods have been presented by employing such properties [3][4][5]. Unfortunately, these properties usually change as texture images undergo affine transforms. Chang et al. [12] deal with texture discrimination by projective invariants. However their methods are based on the use of cross ratios and are only suitable for textures with strong line properties. Furthermore, selection of feature points still remains a problem.

In this paper, we focus on structural textures and investigate affine invariant texture classification by employing structural properties. In Section 2, we give the mathematical details of the area ratio as affine invariants. In Section 3, an area-ratio map utilizing these invariants is introduced to give a description of texture. Histogram of this map is constructed for affine invariant texture classification. In Section 4, experiments are described along with the presentation of the results. Conclusions are presented in Section 5.

2. Affine invariant structural properties

From the structural point of view, texture is composed of texture elements called texels, which are arranged according to certain placement rules. Area of a texel is the area of the region covered by the texel in an image. They do not change with rotation, translation, but change under scaling or skewing transform. Compactness of a texel is invariant to scale, but varies under skewing transform. It has been pointed out that affine transform can be splitted into

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three components based on Singular Value Decomposition, two of which are rotation and the other is skew [8]. Therefore the structural properties mentioned above cannot be directly used for affine invariant texture analysis. In the following, we derive some properties based on texel areas which are invariant to affine transform.

Theorem 1: The ratio of the areas of two triangles does not change under affine transform.

Proof: For 3 non-collinear points $p_m(x_m, y_m) \quad m = 1,2,3$, the area of the triangle formed by the three points is given by

$$S_{\Delta P_1P_2P_3} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} \qquad (1)$$

Given an affine transform represented by matrix $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ and the translation factor $\begin{vmatrix} a_{13} \\ a_{23} \end{vmatrix}$, let point $P'_m(x'_m, y'_m)$ denote the affine transformed version of $p_m(x_m, y_m)$ They have

$$S'_{\Delta P'_1P'_2P'_3} = \frac{1}{2}\begin{vmatrix} x'_1 & x'_2 & x'_3 \\ y'_1 & y'_2 & y'_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a_{11}x_1 + a_{12}y_1 + a_{13} & a_{11}x_2 + a_{12}y_2 + a_{13} & a_{11}x_3 + a_{12}y_3 + a_{13} \\ a_{21}x_1 + a_{22}y_1 + a_{23} & a_{21}x_2 + a_{22}y_2 + a_{23} & a_{21}x_2 + a_{22}y_2 + a_{23} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= |a_{11}a_{22} - a_{12}a_{21}| S_{\Delta P_1P_2P_3} \qquad (2)$$

The ratio of the areas of triangle $\Delta P_1P_2P_3$ and its transformed version $\Delta P'_1P'_2P'_3$ is thus given by

$$\frac{S'_{\Delta P'_1P'_2P'_3}}{S_{\Delta P_1P_2P_3}} = |a_{11}a_{22} - a_{12}a_{21}| \qquad (3)$$

Suppose there is another triangle $\Delta Q_1Q_2Q_3$ formed by $Q_m(x_m, y_m), \quad m = 1,2,3$. Similarly, we have:

$$\frac{S'_{\Delta Q'_1Q'_2Q'_3}}{S_{\Delta Q_1Q_2Q_3}} = |a_{11}a_{22} - a_{12}a_{21}| \qquad (4)$$

By combining Equation (3) and (4), we obtain:

$$\frac{S_{\Delta P_1P_2P_3}}{S_{\Delta Q_1Q_2Q_3}} = \frac{S'_{\Delta P'_1P'_2P'_3}}{S'_{\Delta Q'_1Q'_2Q'_3}} \qquad (5)$$

This indicates that the ratio of the areas of two triangles is invariant to affine transform. Since any closed planar shape can be approximated by triangle elements, the following conclusion can be easily drawn:

Theorem 2: The ratio of the areas of two closed planar shapes is invariant to affine transform.

These properties cannot be directly used for affine invariant texture analysis as they require the correspondences between texels and their transformed versions that are difficult to establish. In the following section we introduce an area ratio map to deal with this problem. A histogram of this map is established to present the similarity measurement.

3. Affine invariant histogram based on area ratio map

For our algorithm, the first step is to segment an image into uniform intensity regions. This is important for classification, as it directly affects the classification accuracy. Texture images often contain noise. Furthermore, presence of texture surfaces often shows non-uniform luminance and contrast. In such cases, a threshold that works well in one area of the image might work poorly in other areas. For this reason, adaptive thresholding [13] is chosen (It must be pointed that texel extraction is a very difficult and challenging problem, more sophisticated segmentation techniques may be exploited but this is not the focus of this paper.). Segmentation often produces many small holes in texel regions. This may cause errors in classification. Morphological operations are performed to eliminate these small holes. After these operations, texels are extracted. The performance of these operations is illustrated in Fig 1. The texels shown in Fig.1(d) are directly extracted from Fig.1(b) (here we take white regions as texels). The texels shown in Fig.1(e) are derived from Fig.1(c) that is obtained from Fig.1(b) after performing morphological operation. We can see that Fig.1(e) is less noisy than Fig.1(d) and the performance of texels extraction is greatly improved.

Suppose that there are n texels in a texture image. An area ratio map matrix can be defined as follows (illustrated in Figure 2):

At the jth column and ith row is the ratio of areas of texel i and texel j represented by

$$r(i, j) = \frac{area \quad of \quad texel\ i}{area \quad of \quad texel\ j} \qquad (6)$$

Thus a matrix $r(i, j) \quad i, j = 1,2...n$ is established. The size of the ratio matrix is $n \times n$.

Notice that there are only $\frac{(n-1)(n-2)}{2}$ independent elements in this matrix. The probability of texel pairs having value $r(i, j)$ is defined as follows:

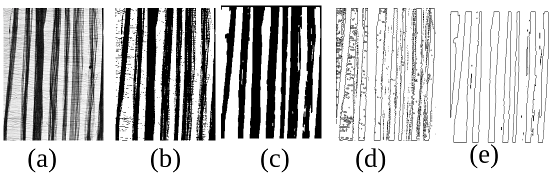

Figure 1 An example illustrating the process of texel extraction.
(a): original texture image (d051) selected from Brodatz album.
(b): segmented image with the adaptive threshold method
(c): binary image after performing morphological operations on (b) including dilation and erosion.
(d): texels extracted from (b).
(e): texels extracted from (c) (less noisy than (d)).

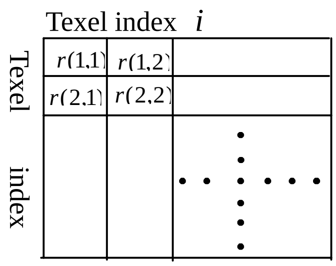

Figure 2. Illustration of area-ratio map

$$f_r = \frac{k_r}{s}, \quad s = \frac{n(n+1)}{2} \tag{7}$$

where Kr is the total number of texel pairs with area ratio r and s is the total number of all possible texel pairs. An area ratio histogram $h(r) = f_r$ is thus established based on the area-ratio matrix. It is obvious that these histograms are invariant to affine transforms and can be used as affine invariant texture features (it is important to note that the calculation of these features do not require the correspondence between texels).

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4. Experimental results

To show that histograms based on area ratios are invariant to affine transform and can be used for affine invariant texture classification, we have conducted a number of experiments. The similarity of the histograms is calculated as follows:

Let $C(k)$ $(k = 1,2,3 \cdots n)$ be the cumulative distribution function derived from histogram $h(i)$,

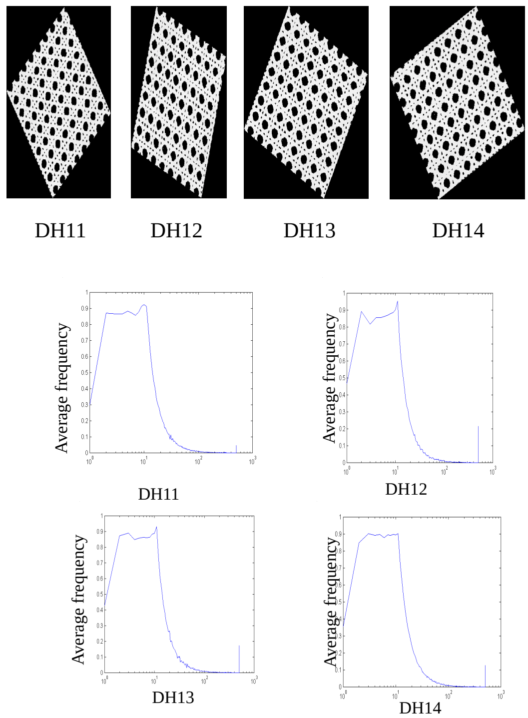

Figure 3. The first row is the four affine transformed versions of Brodatz texture D012. The second and the third row is the coresponding area-ratio histograms

i.e., $C(k) = \sum_{i=1}^{k} h(i)$. Let the cumulative distribution functions of two histograms be $C_1(K)$ and $C_2(K)$. The distance between the two histograms is defined as $D = \sum |C_1(k) - C_2(k)|$. We use the distance D to represent the similarity between two underlying textures. The smaller the value of D, the more similar the two textures.

4.1. Intra- and inter-class similarities

Figure 3 and Figure 4 show four affine transformed

Table 1 Similarity of the histograms shown in Figure 3 and Figure 4

Distance	DH11	DH12	DH13	DH14	DH21	DH22	DH23	DH24
DH11	0	0.096	0.188	0.208	1.265	1.261	1.289	1.295
DH12	0.096	0	0.092	0.112	1.169	1.165	1.193	1.199
DH13	0.188	0.092	0	0.020	1.078	1.074	1.102	1.108
DH14	0.208	0.112	0.020	0	1.058	1.054	1.082	1.088
DH21	1.265	1.169	1.078	1.058	0	0.004	0.024	0.030
DH22	1.261	1.165	1.074	1.054	0.004	0	0.028	0.034
DH23	1.289	1.193	1.102	1.082	0.024	0.028	0	0.007
DH24	1.295	1.199	1.108	1.088	0.030	0.034	0.007	0

Table 2 Confusion matrix for 8 textures shown in Figure 5

Reference texture	Matches with (%)							
	D005	D012	D020	D051	D056	D074	D075	D104
D005	80	0	0	0	20	0	0	0
D012	0	100	0	0	0	0	0	0
D020	0	0	100	0	0	0	0	0
D051	0	0	0	100	0	0	0	0
D056	0	0	0	0	100	0	0	0
D074	10	0	0	0	0	90	0	0
D075	0	0	0	0	0	0	100	0
D104	0	0	0	0	0	0	0	100
Average classification accuracy=96.25%								

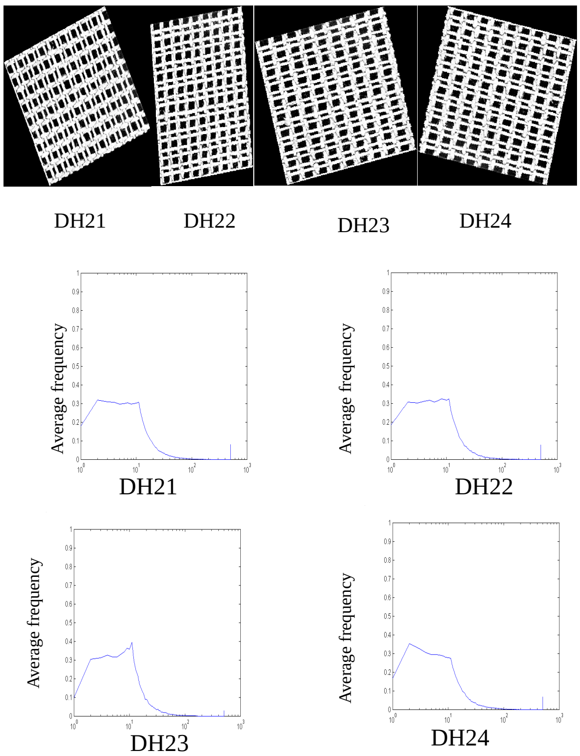

Figure 4 The first row is the four affine transformed versions of Brodatz texture D020, and the second row and the third row are the corresponding area-ratio histograms.

versions of texture D012 and four affine transformed versions of texture D020 with the corresponding area ratio histograms. Table 1 tabulates the similarity of these histograms. It can be seen that the similarity value (measured by distance D) is very low within the same textures, whereas it is much higher between different textures. This indicates that the area ratio histograms can be used for affine invariant texture recognition.

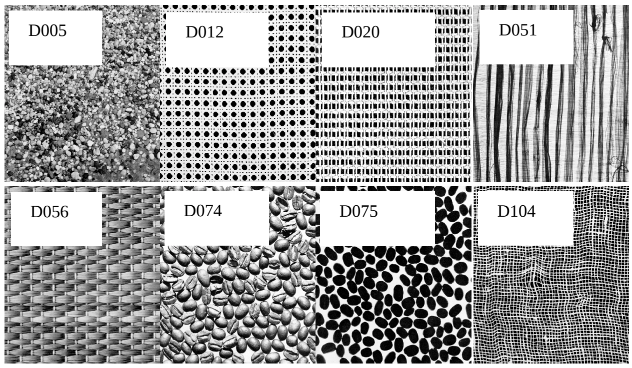

Figure 5 Eight natural textures from Brodatz database used for texture classification

4.2. Texture classification

In order to test the performance of our

algorithm for affine invariant texture classification, 8 natural textures from the Brodatz texture album are used as shown in Figure 5. Each texture is randomly affine-transformed into 30 versions (10 for training and 20 for testing). Thus a total of 240 texture images are constructed. The area ratio histogram is used to characterize texture properties. The k-nearest neighbour algorithm is employed for texture classification. Table 2 shows the results of texture classification. From the confusion matrix, one can see that a classification accuracy of 100% is achieved for six texture classes (D051, D056, D075, D104, D020 and D012). The lowest classification accuracy of 80% is shown for D005. This is because that there exist many small texels in D005. When this image undergoes affine transform, these small elements do not strictly satisfy the affine mathematical model due to quantization errors. This strongly affects the calculation of area ratios of these texels, which in turn results in the misclassification. Therefore our method may not be suitable for micro textures where texels are not easily identifiable.

4.3. Noise robustness

This additional experiment is performed to investigate the noise robustness of the proposed affine invariant texture features. Three most widely used noise models (Gaussian white noise, Salt&Pepper noise and Speckle noise) are simulated. Each texture image is added with the three noises at different SNR values. Examples of texture images corrupted by Gaussian white noise are shown in Figure 6. Here the definition of SNR is described as:

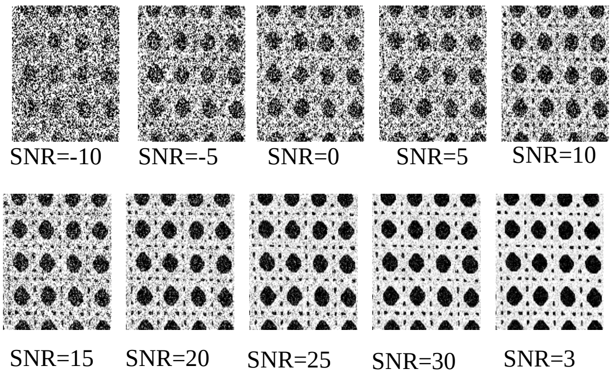

Figure 6. Texture images from Brodatz database corrupted by Gaussian white noise at different SNR values

$$SNR = 20\log\frac{Es}{En} \tag{8}$$

where Es denotes the energy function of the original image defined by $\int s^2(x,y)dxdy$ and En the energy function of noise signal defined by $\int n^2(x,y)dxdy$. The classifier is trained with noise-free images and tested with the noisy images. No noisy images are used for training. The

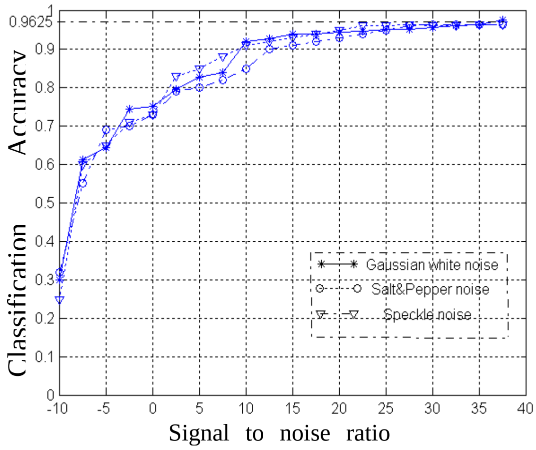

Figure 7. Noise robustness of the affine invariant texture features

performance of our algorithm under these noise models at different noise level is shown in Figure 7. We can see that our algorithm exhibits almost the same performance under different noise models and produce promising correct recognition rates around 80% at very low signal to noise ratio (SNR=5). This indicates that our method is quite robust to noise.

5. Conclusion

In this paper, we have presented a new method for texture classification that is invariant to affine transformation. We derive the affine invariant structural properties and introduce an area-ratio map by utilizing these properties to characterize texture images. Histogram based on this map is constructed and successfully used for affine invariant texture classification. The results of the experiments with natural textures form Brodatz album have shown that the algorithm performs well. Furthermore, our algorithm can also work well with very noisy texture images. Notice that this method is based on texel extraction (although this is not the focus of this paper), so it is more suitable for those textures where the texels can be easily defined. Since that the surface of each texel can also be considered as a small texture image, these surfaces describe the detailed texture information of the whole image in a smaller scale.

This feature should be taken into account in future research to give a more robust and precise accuracy.

6. Reference

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