# Nontermination in Type Class Inference

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# Introduction: Type Class Inference

Informally speaking

- Type Class Inference $=$ Hindley-Milner $+$ Instance Resolution

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- Type Class Inference $=$ Hindley-Milner $+$ Instance Resolution
- Instance Resolution $=$ Context Reduction $+$ Evidence Construction

# Introduction: Example

```
class Eq a where
  eq :: Eq a => a -> a -> Bool

instance Eq a, Eq b => Eq (a, b) where
  eq (x1, y1) (x2, y2) = and (eq x1 x2) (eq y1 y2)

instance => Eq Char where
  eq = primtiveCharEq

-- test :: Eq (Char, Char) => Bool
test = eq (’a’, ’b’) (’c’, ’d’)
```

# Introduction: One possible translation

```
data Eq a where
  CEq :: (a -> a -> Bool) -> Eq a

eq :: Eq a -> (a -> a -> Bool)
eq (CEq m) = m

f :: Eq a, Eq b -> Eq (a, b)
f d1 d2 = CEq q
 where q (x1, y1) (x2, y2) =
         and (eq d1 x1 x2) (eq d2 y1 y2)

g :: Eq Char
g = CEq primtiveCharEq

test = eq d (’a’, ’b’) (’c’, ’d’)
-- some d :: Eq (Char, Char)
```

# Introduction: Context Reduction

- Given

  ```
  f :: Eq a, Eq b -> Eq (a, b)
  g :: Eq Char
  ```

  How to *automatically* construct

  `d ::  Eq (Char, Char)` ?

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- Rewrite Rules $\Phi$ on Multiset:

  $\{\ldots\texttt{Eq (a, b)}\ldots\} \rightarrow_f \{\ldots\texttt{Eq a, Eq b}\ldots\}$

  $\{\ldots\texttt{Eq Char}\ldots\} \rightarrow_g \{\ldots\}$

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- So `d = f g g`

# Context Reduction

$\Phi \vdash \{\texttt{Eq (Char, Char)}\} \rightarrow_f \{\texttt{Eq Char,Eq Char}\} \rightarrow_g$
$\{\texttt{Eq Char}\} \rightarrow_g \emptyset$
Thus `d = f g g`

- Evidence construction seems to rely on termination of context reduction
- What happen if we have a non-terminating reduction?

# Context Reduction: Nontermination

- Example[1]:

```
instance Data SizeD t => Size t where ...
instance Sat (c Char) => Data c Char where ...
instance Size t => Sat (SizeD t) where ...
```

- Corresponding rules($\Phi$):

$\{\ldots\texttt{Size t}\ldots\} \rightarrow_a \{\ldots\texttt{Data SizeD t}\ldots\}$

$\{\ldots\texttt{Data c Char}\ldots\} \rightarrow_b \{\ldots\texttt{Sat (c Char)}\ldots\}$

$\{\ldots\texttt{Sat (SizeD t)}\ldots\} \rightarrow_c \{\ldots\texttt{Size t}\ldots\}$

[1] from R. Lämmel & S.P. Jones's Scrap your boilerplate with class

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- How to construct an evidence

  `d ::  Data SizeD Char`?

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  $\{\ldots\texttt{Sat (SizeD t)}\ldots\} \rightarrow_c \{\ldots\texttt{Size t}\ldots\}$

- How to construct an evidence `d :: Data SizeD Char`?

- Let's try to reduce `Data SizeD Char`:

  $\Phi \vdash \{\texttt{Data SizeD Char}\} \rightarrow_b$
  $\{\texttt{Sat (SizeD Char)}\} \rightarrow_c \{\texttt{Size Char}\} \rightarrow_a$
  $\{\texttt{Data SizeD Char}\} \rightarrow_b \cdot \rightarrow_c \cdot \rightarrow_a \ldots$

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# Nontermination

- What can we do when context reduction diverge?

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- What can we do when context reduction diverge?
- Cycle detection(tie the knot)
  $\{\texttt{Data SizeD Char}\} \to_b \{\texttt{Sat (SizeD Char)}\} \to_c \{\texttt{Size Char}\} \to_a \{\texttt{Data SizeD Char}\} \to_b \cdot \to_c \cdot \to_a \ldots$
- Given:

```
a :: Data SizeD t -> Size t
b :: Sat (c Char) -> Data c Char
c :: Size t -> Sat (SizeD t)
```

We have:

```
d :: Data SizeD Char
d = b (c (a d))
```

# Nontermination

What if the context reduction is diverging without forming any cycle?

```
data Nested a where
 Nil :: Nested a
 Cons :: a -> Nested [a] -> Nested a

instance Eq a, Eq (Nested [a]) => Eq (Nested a) where
  eq Nil Nil = True
  eq (Cons a as) (Cons b bs) = eq a b && eq as bs
```

```
{...Eq (Nested a)...} → {...Eq a, Eq (Nested [a])...}
{Eq (Nested Char)} →
{Eq Char,Eq (Nested [Char])} →
{Eq Char,Eq [Char],Eq (Nested [[Char]])}...
```

# Nontermination

What if the context reduction is diverging without forming any cycle?

- Context reduction seems too *earger*.
- How to have *lazy* context reduction?
- A change of perspective:
  Rewriting on multiset:
  $\{\ldots\texttt{Eq (a, b)}\ldots\} \to_f \{\ldots\texttt{Eq a}, \texttt{Eq b}\ldots\}$
  $\{\ldots\texttt{Eq Char}\ldots\} \to_g \{\ldots\}$
  Rewriting on first order term:
  $\texttt{Eq (a, b)} \to f\,(\texttt{Eq a})\,(\texttt{Eq b})$
  $\texttt{Eq Char} \to g$
- Constructing `d :: Eq (Char, Char)`
  $\texttt{Eq (Char, Char)} \to f\,(\texttt{Eq Char})\,(\texttt{Eq Char}) \to$
  $f\ g\,(\texttt{Eq Char}) \to f\ g\ g$

# Summary and Further Works

- Evidence construction process is a kind of rewriting process
- Knowing termination behavior in advance, we may be able to rewrite eagerly/lazily
- **Next Step** Extend the current cycle detection techniques to obtain evidence(statically) for non-obvious “looping” example
- **Long Term Goal** Explore the connection between the evidence that gives rise to infinite rewrite process and the notion of productivity in Katya’s Structural Resolution