# Relating small ontologies

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## 1 Introduction

We are interested in exploring representation change in mathematics; work by [5] suggests that the structure of metaphor plays an important cognitive role in the development of mathematical theories. In their work, metaphor is taken to involve "grounded, inference-preserving cross-domain mappings" [5, p 6].

One way to think about such mappings builds on work by Goguen [2, 1], who proposed related notions of (semiotic and frame) *morphism* to express relations that can hold between some statements or signs making a statement about some domain, and some other statements or signs related to a different domain.

One case is where the statements may be in a simple ontology language, but where there is a series of related ontologies, each describing different but also related domains. Goguen's *frame morphisms* are suggestive in giving a framework for describing the components of this situation.

## 2 A Case Study

To illustrate the approach, we look at the central example in Lakatos's famous reconstruction of the history surrounding Euler's formula [4]. It is of course interesting to try to model the evolving theories that crop up during the refinement of the ideas involved: Lakatos's suggestions allow certain basic operations of theories to be computationally realised, e.g. as described in [6]. Here, however, we are interested in analysing what goes on in the original, flawed, *procedural* proof, given in terms of a set of steps intended to preserve certain properties.

Steps in the argument involve carrying out operations on a system of connected faces, considered to lie on a flat plane. A sequence of miniature ontologies describes the state of this system at different stages.

The approach via frame morphisms suggests that we relate each ontology to a geometrical (or perhaps combinatorial) object. An operation on the ontology, e.g. consisting of *the removal of a point and two lines* can be defined over the syntax of the ontology. There should then be a related morphism, in the opposite direction, between the corresponding geometrical objects: in this case the simple embedding of one system of faces into another extended system works well.

The ontology statements can be taken to be statements in first-order logic. The semantics involved is not the standard Tarskian reading, however, since we are invoking the notion of a single canonical model. An advantage of the approach via frame morphisms is that we are not tied to a particular logic, and can thus happily make use in this way of aspects of closed world reasoning which are natural in this context.

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The subsequent history of the mathematics of this example involves the field of Algebraic Topology, which is full of "inference-preserving cross-domain mappings" that relate the algebraic and topological domains. The analysis above brings out aspects of the reasoning involved in going from a given 3-dimensional polygon to the more combinatoric graph-like reasoning involved elsewhere, and we can claim that the intuitions involved in this example are grounded in manipulations of packing cases, and so on. Related work in diagrammatic reasoning, such as [3], suggests that such forms of reasoning are found more intuitive than conventional syntactic proofs. An open question here is to what extent such grounding might be involved in the much more abstract developments of Algebraic Topology.

## REFERENCES

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